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The magic of four zero neutrino Yukawa textures *

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Four is the maximum number of texture zeros allowed in the Yukawa coupling matrix of three massive neutrinos. These completely fix the high scale CP violation needed for leptogenesis in terms of that accessible at laboratory energies. $\mu\tau$ symmetry drastically reduces such allowed textures. Only one form of the light neutrinos mass matrix survives comfortably while another is marginally allowed.

Keywords: Neutrino Mass; Texture Zeros; Flavor symmetry

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1. Introduction

There is something magical about four zero Yukawa textures. After earlier success in the quark sector[1], it is proving useful with leptons, as explained in the abstract. Within the type-I seesaw framework and in the weak basis of mass diagonal charged leptons and heavy right chiral neutrinos, the results stated above were derived[2] leading to a highly constrained and predictive scheme[2,3]. We shall discuss the effect of the imposition of $\mu\tau$ symmetry[4] on this scheme.

The light neutrino mass matrix in the usual notation is

$$M_\nu \simeq -m_D M_R^{-1} m_D^T, \tag{1}$$

with $O(M_R) \gg O(m_D)$. M_ν diagonalizes as under

$$U^\dagger M_\nu U^* = M_d^\nu = \mathrm{diag}(m_1,\ m_2,\ m_3), \tag{2}$$

$m_{1,2,3}$ being real and positive. Our PMNS parametrization is

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13}e^{-i\delta_D} \\ 0 & 1 & 0 \\ s_{13}e^{i\delta_D} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_M} & 0 & 0 \\ 0 & e^{i\beta_M} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

where $c_{ij} \equiv \cos\theta_{ij},\ s_{ij} \equiv \sin\theta_{ij}$ and $\delta_D,\ \alpha_M,\ \beta_M$ are the one Dirac phase and two Majorana phases respectively. In our basis, $M_\ell = \mathrm{diag}(m_e,\ m_\mu,\ m_\tau)$ and

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$M_R = \text{diag}(M_1,\ M_2,\ M_3)$, all mass eigenvalues being real and positive. The Casas-Ibarra [5] form for m_D which equals the neutrino Yukawa coupling matrix times the relevant Higgs VEV, is

$$m_D = iU\sqrt{M_\nu^d}R\sqrt{M_R}, \tag{4}$$

where R in general is an unknown complex orthogonal matrix: $R^TR = RR^T = I$. The best fit experimental numbers, needed by us, appear in Table 1. Loosely, $R = \frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} \simeq 3.2\times 10^{-2}$, $\theta_{23} \simeq \frac{\pi}{4}$, $\theta_{12} \simeq \sin^{-1}\frac{1}{\sqrt{3}}$ and θ_{13} is small. We assume no massless neutrino, i.e. $\det M_\nu \neq 0$.

Table 1. Best-fit experimental numbers from [6]

Quantity	Experimental values
$\Delta m^2_{21} = m_2^2 - m_1^2$	$7.59 \pm 0.20 \left({+0.61 \atop -0.69}\right) \times 10^{-5}\ eV^2$
$\Delta m^2_{32} = m_3^2 - m_2^2$	$-2.40 \pm 0.11 \left({+0.37 \atop -0.36}\right) \times 10^{-3}\ eV^2$(inverted)
	$2.51 \pm 0.12 \left({+0.39 \atop -0.36}\right) \times 10^{-3}\ eV^2$(normal)
θ_{12}	$34.4 \pm 1.0 \left({+3.2 \atop -2.9}\right)^\circ$
θ_{23}	$42.3^{+5.3}_{-2.8} \left({11.4 \atop -7.1}\right)^\circ$
θ_{13}	$< 13.2^\circ$

2. Four zero Yukawa textures and $\mu\tau$ symmetry

It is more natural to attribute ab initio textures to m_D, which appears in the Lagrangian rather than to the derived m_ν. 72 allowed four zero textures in m_D have been classified into[2] two categories: (A) 54 with one pair of vanishing conjugate off diagonal elements in M_ν and (B) 18 with two zeros in one row and one each in the other two (k, l say) obeying $\det(\text{cofactor}(M_\nu)_{kl}) = 0$. For all these, the R matrix of Eq. (4) has been reconstructed in terms of the element of U, $M_\nu^{\rm d}$ and M_R. Consequently, all phases of R are given in terms of δ_D, α_M and β_M which completely determine all phases in m_D including those responsible for leptogenesis.

Elements of m_D and M_R are required by $\mu\tau$ symmetry to remain invariant under the interchange $\nu_\mu \leftrightarrow \nu_\tau$, $N_\mu \leftrightarrow N_\tau$. The seesaw formula immediately implies a custodial $\mu\tau$ symmetry in M_ν itself, leading to $\theta_{23} = \frac{\pi}{4}$, $\theta_{13} = 0$. Further, the number of four zero textures allowed in m_D is drastically reduced[7]. Only two each

are allowed in categories (A) and (B) both leading to the same $M_\nu^{(A)}$ or $M_\nu^{(B)}$:

$$M_\nu^{(A)} = m \begin{pmatrix} k_1^2 e^{2i\alpha} + 2k_2^2 & k_2 & k_2 \\ k_2 & 1 & 0 \\ k_2 & 0 & 1 \end{pmatrix} \qquad M_\nu^{(B)} = m' \begin{pmatrix} l_1^2 & l_1 l_2 e^{i\beta} & l_1 l_2 e^{i\beta} \\ l_1 l_2 e^{i\beta} & l_2^2 e^{2i\beta} + 1 & l_2^2 e^{2i\beta} \\ l_1 l_2 e^{i\beta} & l_2^2 e^{2i\beta} & l_2^2 e^{2i\beta} + 1 \end{pmatrix} \tag{5}$$

Here m and m' are overall mass scales, $k_1,\ k_2,\ l_1,\ l_2$ are real parameters and α, β are phases.

Turning to θ_{12} and R, one can derive that

$$\begin{aligned} R &= 2(X_1^2 + X_2^2)^{1/2} [X_3 - (X_1^2 + X_2^2)^{1/2}]^{-1}, \\ \tan 2\theta_{12} &= \frac{X_1}{X_2}. \end{aligned} \tag{6}$$

The X's of Eq. (6) are given for $M_\nu^{(A)}$ as

$$\begin{aligned} X_1^{(A)} &= 2\sqrt{2} k_2 [(1 + 2k_2^2)^2 + k_1^4 + 2k_1^2 (1 + 2k_2^2)\cos 2\alpha]^{1/2}, \\ X_2^{(B)} &= 1 - k_1^4 - 4k_2^4 - 4k_1^2 k_2^2 \cos 2\alpha, \\ X_3^{(A)} &= 1 - 4k_2^4 - k_1^4 - 4k_1^2 k_2^2 \cos 2\alpha - 4k_2^2. \end{aligned} \tag{7}$$

For $M_\nu^{(B)}$, they are given by

$$\begin{aligned} X_1^{(B)} &= 2\sqrt{2} l_1 l_2 [(l_1^2 + 2l_2^2)^2 + 1 + 2(l_1^2 + 2l_2^2)\cos 2\beta]^{1/2}, \\ X_2^{(B)} &= 1 + 4l_2^2 \cos 2\beta + 4l_2^4 - l_1^4, \\ X_3^{(B)} &= 1 - (l_1^2 + 2l_2^2)^2 - 4l_2^2 \cos 2\beta. \end{aligned} \tag{8}$$

One can further impose the requirement of tribimaximal mixing, i.e. $\theta_{12} = \sin^{-1}\frac{1}{\sqrt{3}} \simeq 35.2^\circ$, which needs $(M_\nu)_{11} + (M_\nu)_{12} = (M_\nu)_{22} + (M_\nu)_{23}$. For $M_\nu^{(A)}$, α is then immediately fixed at $\pi/2$ and $k_1 = (2k_2^2 + k_2 - 1)^{1/2}$. For $M_\nu^{(B)}$, β is then immediately fixed at $\cos^{-1}(l_1/4l_2)$ and $l_2 = (1 - l_1^2)^{1/2}/2$. For category (A) R equals $3(k_2 - 2)/(k_2 + 1)$, while for category (B) it becomes $3l_1^2/2(1 - 2l_1^2)$.

3. Phenomenology

Experimentally fitted values of R and $\tan 2\theta_{12}$ can be matched with the expression in Eq. (6) - Eq. (8). For category (A), only the spectrum with inverted ordering is found to be allowed. But no common allowed parameter ranges are found for $(k_1,\ k_2,\ \alpha)$ in the 1σ intervals of $R = -(2.88 - 3.37) \times 10^{-2}$ and $\theta_{12} = 33.15^\circ - 35.91^\circ$. The 3σ intervals $R = -(2.46 - 3.99) \times 10^{-2}$ and $\theta_{12} = 30.66^\circ - 39.23^\circ$ allow a thin strip (Fig. 1) in the $k_1 - k_2$ plane with $89^\circ \le \alpha \le 90^\circ$ and $2.0 < k_1 < 5.3$, $1.2 < k_2 < 3.7$. The additional constraint of tribimaximal mixing, when α must be exactly $\pi/2$, confines k_2 to the range $1.95 \le k_2 \le 1.97$. Improved experimental errors may thus rule out this category completely.

Only the normally ordered spectrum is found to be allowed for category (B). But now there are significant allowed regions in the $l_1 - l_2$ plane both for 1σ and 3σ

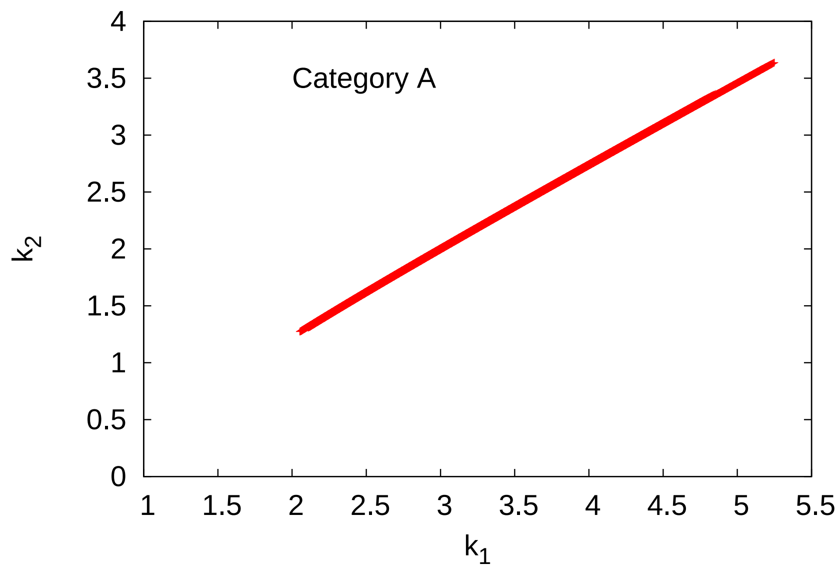

Fig. 1. Variation of k_1 and k_2 in category A with $\mu\tau$ symmetry over the 3σ allowed ranges of R and θ_{12}.

intervals R and θ_{12}. Specifically for the 3σ intervals $R = (2.52 - 4.07) \times 10^{-2}$ and $\theta_{12} = 30.66° - 39.23°$, two allowed branches appear (Fig.2) with β in the ranges 87° to 90°, $0.1 < l_1 < 0.55$ and $0.6 < l_2 < 0.76$. The further imposition of tribimaximal mixing forces the only one free variable left, namely l_1, to be within the range $0.11 \leq l_1 \leq 0.15$.

4. Radiative lepton flavor violating decay and leptogenesis

With $\alpha > \beta$ and $l_1 = l_e, l_2 = l_\mu, l_3 = l_\tau$, the branching ratio for the decay $l_\alpha \to l_\beta \gamma$ can be written in mSUGRA scenarios (with universal scalar masses at a GUT scale $M_\chi \sim 2 \times 10^{16}$ GeV) as

$$\mathrm{BR}(l_\alpha \to l_\beta \gamma) \propto \mathrm{BR}(l_\alpha \to l_\beta \nu \bar{\nu}) |(m_D L m_D^\dagger)_{\alpha\beta}| \tag{9}$$

with

$$L_{kl} = \ln \frac{M_X}{M_k} \delta_{kl}, \tag{10}$$

M_k being the mass of the kth heavy right chiral neutrino N_k. Now $\mathrm{BR}(\tau \to \mu\gamma)$ vanishes for category (A) since $(M_\nu^{(A)})_{23} = 0$. Otherwise, the allowed textures in both categories have $(M_\nu)_{13} = (M_\nu)_{12} \neq 0$ and lead to nonzero $\mathrm{BR}(\tau \to e\gamma)$ and $\mathrm{BR}(\mu \to e\gamma)$ but with the relation

$$\frac{\mathrm{BR}(\tau \to e\gamma)}{\mathrm{BR}(\mu \to e\gamma)} \simeq \frac{\mathrm{BR}(\tau \to \mathrm{e}\nu_\mathrm{e}\bar{\nu}_\mathrm{e})}{\mathrm{BR}(\mu \to \mathrm{e}\nu_\mu\bar{\nu}_\mu)} \simeq 0.178. \tag{11}$$

For leptogenesis, the flavor dependent lepton asymmetry in the standard notation is given for the Minimal Supersymmetric Standard Model by

$$\begin{aligned}\varepsilon_i^\alpha &= \frac{\Gamma(N_i \to \phi \bar{l}_\alpha) - \Gamma(N_i \to \phi^\dagger l_\alpha)}{\sum_\beta [\Gamma(N_i \to \phi \bar{l_\beta}) + \Gamma(N_i \to \phi^\dagger l_\beta)]} \\ &\simeq \frac{g^2}{16\pi M_W^2} \frac{1}{(m_D^\dagger m_D)_{ii}} \sum_{j \neq i} \left[\mathcal{I}_{ij}^\alpha f\left(\frac{M_j^2}{M_i^2}\right) + \mathcal{J}_{ij}^\alpha \left(1 - \frac{M_j^2}{M_i^2}\right)^{-1} \right], \qquad (12)\end{aligned}$$

$$\mathcal{I}_{ij}^\alpha = \mathrm{Im}(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}(m_D^\dagger m_D)_{ij} = -\mathcal{I}_{ji}^\alpha, \qquad (13)$$

$$\mathcal{J}_{ij}^\alpha = \mathrm{Im}(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}(m_D^\dagger m_D)_{ji} = -\mathcal{J}_{ji}^\alpha, \qquad (14)$$

$$f(x) = \sqrt{x}\left[\frac{2}{1-x} - \ln\frac{1+x}{x}\right]. \qquad (15)$$

The flavor independent lepton asymmetry is

$$\varepsilon_i = \sum_\alpha \varepsilon_i^\alpha = \frac{g^2}{16\pi M_W^2} \frac{1}{(m_D^\dagger m_D)_{ii}} \sum_{j \neq i} \left[(m_D^\dagger m_D)_{ij}\right]^2 f\left(M_j^2/M_i^2\right). \qquad (16)$$

For $M_1 << M_{2,3}$, $f(M_{2,3}^2/M_1^2) \simeq -3M_1/M_{2,3}$. Table 2. summarizes our statement on the leptogenesis parameters including the effective mass $\widetilde{m_1}^\alpha = |(m_D)_{\alpha 1}|^2/M_1$ for the washout of the α-flavor asymmetry

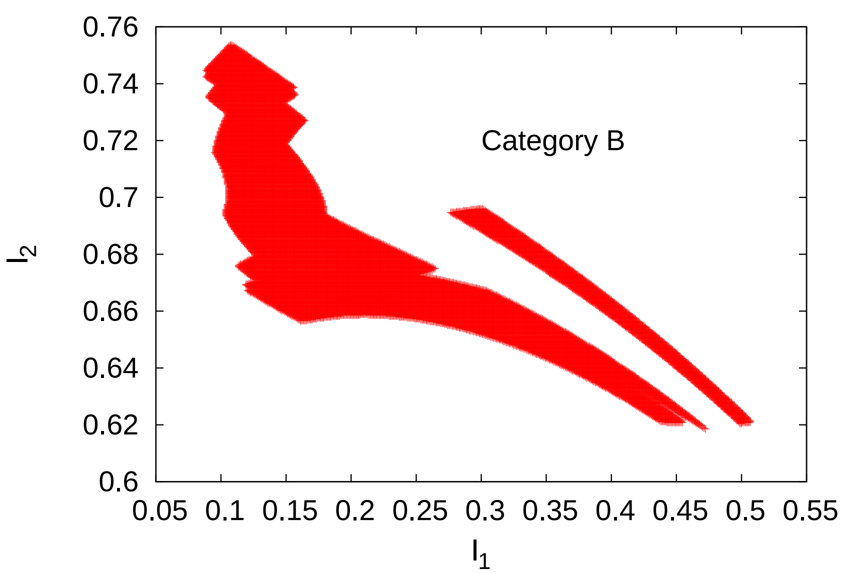

Fig. 2. Variation of l_1 and l_2 in Category B for the 3σ allowed ranges of R and θ_{12}

Table 2. Leptogenesis Table

configuration	$\mathcal{I}_{ij}^{\alpha}$	$\mathcal{J}_{ij}^{\alpha}$	$\widetilde{m_1}^e$	$\widetilde{m_1}^\mu$	$\widetilde{m_1}^\tau$
$m_D^{(1)}$	$\mathcal{I}_{12}^e = \mathcal{I}_{13}^e \neq 0$, rest zero	0	nonzero	0	0
$m_D^{(2)}$	–do–	0	nonzero	0	0
$m_D^{(3)}$	$\mathcal{I}_{12}^\mu = \mathcal{I}_{13}^\tau \neq 0$, rest zero	0	nonzero	nonzero	equals $\widetilde{m_1}^\mu$
$m_D^{(4)}$	$\mathcal{I}_{13}^\mu = \mathcal{I}_{12}^\tau \neq 0$, rest zero	0	nonzero	nonzero	equals $\widetilde{m_1}^\mu$

5. Deviation due to RG running

Suppose $\mu\tau$ symmetry in the neutrino sector is imposed at a high scale $\Lambda \sim 10^{12}$ GeV. The neutrino mass matrix elements can then be evolved down to a laboratory scale $\lambda \sim 10^3$ GeV by RG running. On account of the inequality $m_\tau \gg m_\mu \gg m_e$, $\mu\tau$ symmetry is badly broken in the charged lepton sector. Deviations from $\mu\tau$ symmetry creep into M_ν from loop diagrams with charged lepton internal lines. We keep only m_τ induced terms via $\Delta_\tau \simeq \frac{m_\tau^2}{8\pi^2 v^2}(\tan^2\beta + 1)\ln\left(\frac{\Lambda}{\lambda}\right)$ with $v^2 = v_u^2 + v_d^2$ and $\tan\beta = v_u/v_d$, $v_{u,d}$ being up, down type Higgs VEVs in the MSSM. Then to $O(\Delta_\tau)$, $(M_\nu)_{11}$, $(M_\nu)_{12}$, $(M_\nu)_{21}$ and $(M_\nu)_{22}$ are unchanged but the remaining elements change to $(1-\Delta_\tau)(M_\nu)_{13}$, $(1-\Delta_\tau)(M_\nu)_{31}$, $(1-\Delta_\tau)(M_\nu)_{23}$, $(1-\Delta_\tau)(M_\nu)_{32}$ and $(1-2\Delta_\tau)(M_\nu)_{33}$. Consequently, θ_{13} can be nonzero and θ_{23} different from $\pi/4$.

One can redo the phenomenology with these changes. For category (A), the 3σ allowed strip in the $k_1 - k_2$ plane gets marginally extended now with $0 \leq \theta_{13}^\lambda \leq 2.7°$, $\theta_{23} \leq 45°$ while the inverted ordering is retained and the normal one excluded. For category (B), the 3σ allowed branches in the $l_1 - l_2$ plane are enhanced a bit more with $0 \leq \theta_{13}^\lambda \leq 0.85°$, $\theta_{23} \geq 45°$ while retaining the normal ordering and excluding the inverted one.

6. Conclusion

(1) Just four neutrino Yukawa textures with four zeros are compatible with $\mu\tau$ symmetry, leading to only two forms for the light neutrino mass matrix $M_\nu^{(A)}$ and $M_\nu^{(B)}$.
(2) For $M_\nu^{(A,B)}$, 3σ-allowed values of θ_{12} and $R = \Delta m_{21}^2/\Delta m_{32}^2$ admit restricted regions in the parameter space with $M_\nu^{(A)}$ being in some tension with data
(3) The tribimaximal mixing assumption further restricts the the parameters.
(4) Radiative deviations from $\mu\tau$ symmetry yield small values of θ_{13} and can resolve the θ_{23} octant ambiguity ($<$ or $>$ 45°)

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